# Momentum and Impulse

## Questions

**Q1.**

A particle *P* of mass 0.5 kg is moving with velocity 4**j** m $s^{-1}$ when it receives an impulse **I** Ns. Immediately after *P* receives the impulse, the velocity of *P* is (2**i** + 3**j**) m $s^{-1}$.

Find

(a) the magnitude of **I**,

**(4)**

(b) the angle between **I** and **j**.

**(2)**

**(Total for question = 6 marks)**

**Q2.**

Two particles, *P* and *Q*, have masses 2*m* and 3*m* respectively. They are moving towards each other in opposite directions on a smooth horizontal plane when they collide directly. Immediately before they collide the speed of *P* is 4*u* and the speed of *Q* is 3*u*. As a result of the collision, *Q* has its direction of motion reversed and is moving with speed *u*.

(a) Find the speed of *P* immediately after the collision.

**(3)**

(b) State whether or not the direction of motion of *P* has been reversed by the collision.

**(1)**

(c) Find the magnitude of the impulse exerted on *P* by *Q* in the collision.

**(3)**

**(Total for question = 7 marks)**

**Q3.**

A particle $P$, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})$ m s$^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

As a result of the impulse, the direction of motion of $P$ is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of $\lambda$ and $\mu$.

**(Total for question = 9 marks)**

**Q4.**

A particle $P$ of mass 0.5 kg is moving with velocity $(4\mathbf{i} + 3\mathbf{j})$ m s$^{-1}$ when it receives an impulse $\mathbf{J}$ N s. Immediately after receiving the impulse, $P$ is moving with velocity $(-\mathbf{i} + 6\mathbf{j})$ m s$^{-1}$.

(a) Find the magnitude of $\mathbf{J}$.

**(4)**

The angle between the direction of the impulse and the direction of motion of $P$ immediately before receiving the impulse is α°

(b) Find the value of α

**(3)**

**(Total for question = 7 marks)**

**Q5.**

A particle $P$ has mass 0.5 kg. It is moving in the $xy$ plane with velocity $8\mathbf{i}$ m s$^{-1}$ when it receives an impulse $\lambda(-\mathbf{i} + \mathbf{j})$ N s, where $\lambda$ is a positive constant.

The angle between the direction of motion of $P$ immediately before receiving the impulse and the direction of motion of $P$ immediately after receiving the impulse is $\theta$°

Immediately after receiving the impulse, $P$ is moving with speed $4\sqrt{10}\,\text{m s}^{-1}$

Find (i) the value of $\lambda$

(ii) the value of $\theta$

**(8)**

**(Total for question = 8 marks)**

**Q6.**

A particle *A* of mass 3*m* and a particle *B* of mass *m* are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of *A* is *ku* and the speed of *B* is *u*.
Immediately after the collision, the speed of *A* is *v* and the speed of *B* is 2*v*.

The magnitude of the impulse received by *B* in the collision is $\frac{3}{2}$ *mu*.

(a) Find *v* in terms of *u* only.

**(3)**

(b) Find the two possible values of *k*.

**(5)**

**(Total for question = 8 marks)**

**Q7.**

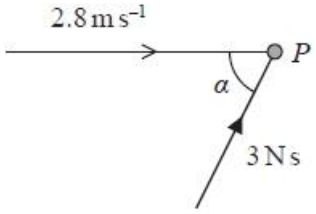



**Figure 2**

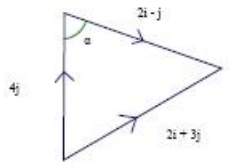
A particle *P* of mass 0.5 kg is moving in a straight line with speed 2.8 m $s^{-1}$ when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of *P* immediately before receiving the impulse and the line of action of the impulse is $\alpha$, where $\tan \alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of *P* immediately after receiving the impulse.

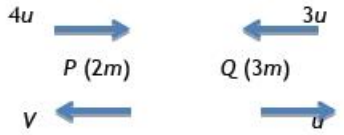
**(Total for question = 5 marks)**

# Mark Scheme – Momentum and Impulse

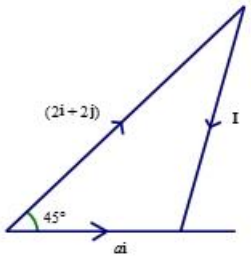
**Q1.**

| Q. | Scheme | Marks | Notes |
|---|---|---|---|
| **a** | $\mathbf{I} = 0.5(\mathbf{2i}+3\mathbf{j}) - 0.5(4\mathbf{j})$ | M1 | Impulse-momentum equation.<br>Dimensionally correct.<br>Condone subtraction in wrong order. |
| | $(= 0.5(2\mathbf{i}-\mathbf{j}))$ | A1 | Correct unsimplified |
| | $\lvert\mathbf{I}\rvert = \frac{1}{2}\sqrt{2^2+1^2}$ | M1 | Correct method for modulus.<br>Follow their **I** |
| | $= \frac{1}{2}\sqrt{5}\,(=1.12)\text{Ns}$ | A1 | 1.1 or better (from correct solution only) |
| | | (4) | |
| | | | |
| **b** | $\tan^{-1}(\pm 2)$ or $\tan^{-1}\left(\pm\frac{1}{2}\right)$ or $\tan\theta = \pm 2$ or $\tan\theta = \pm\frac{1}{2}$ or equivalent | M1 | Correct method for a relevant angle.<br>Follow their **I** |
| | Required angle = 117° (116.6° or better) | A1 | Accept 243° (2.03 rads) |
| | | (2) | |
| **balt** |  | | |
| | $\cos\alpha = \frac{16+5-13}{2\sqrt{5}\sqrt{16}} = \frac{1}{\sqrt{5}}$ | M1 | |
| | Required angle = 117° (116.6°) | A1 | Accept 243° |
| | | (2) | |
| | | | |
| | | [6] | |
| | | | |
| | | | |

**Q2.**

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)** |  <br>$8mu - 9mu = -2mV + 3mu$<br>$V = 2u$ | M1 A1<br>A1 (3) |
| **(b)** | (Has been) reversed | B1 (1) |
| **(c)** | For $Q$: $I = 3m(u - -3u)$<br>$= 12mu$<br>OR:<br>For $P$: $I = 2m(2u - -4u)$<br>$= 12mu$ | M1 A1<br>A1 (3)<br>**OR**<br>M1 A1<br>A1 (3)<br>**7** |
| | **Notes** | |
| **(a)** | M1 for CLM with correct no. of terms, all dimensionally correct, to give an equation in $m$, $u$ and their $V$ only. Condone consistent $g$'s or cancelled $m$'s.<br>First A1 for a correct equation (they may have + $2mV$)<br>Second A1 for $2u$ (must be positive since speed is required) | |
| **(b)** | B1 for '(has been) reversed'. Only available if a correct velocity has been correctly obtained in part (a).<br>B0 for 'changed' ,'direction has changed' , 'yes' | |
| **(c)** | M1 for using Impulse = change in momentum of $Q$ (must have $3m$ in both terms) (M0 if *clearly* adding momenta or if $g$ is included) but condone sign errors.<br>First A1 for $3m(u - -3u)$ or $-3m(u - -3u)$<br>Second A1 for $12mu$ (must be positive since magnitude required)<br>**OR**<br>M1 for using Impulse = change in momentum of $P$ (must have $2m$ in both terms) (M0 if *clearly* adding momenta) but condone sign errors.<br>First A1 for $2m(2u - -4u)$ or $-2m(2u - -4u)$<br>Second A1 for $12mu$ (must be positive since magnitude required)<br>N.B. Allow use of $I = 3m(u - v)$ or $I = 2m(u - v)$ since only magnitude required | |

**Q3.**

| Question | Scheme | Marks | AOs | Notes |
|---|---|---|---|---|
| |  | | | |
| | Momentum of $P$ after impulse = $a\mathbf{i}$ (or $a\mathbf{i}$) | B1 | 2.2a | Correct interpretation of angle of deflection (velocity or momentum a multiple of **i** or **j**) |
| **Either** | Use of $\mathbf{I}=m(\mathbf{v}-\mathbf{u})$: $(\mathbf{I}=)0.5(2a\mathbf{i}-(4\mathbf{i}+4\mathbf{j}))(=(a-2)\mathbf{i}-$ | M1 | 3.3 | Form vector triangle or equation for **v** or their $a\mathbf{i}$ |
| | Use of Pythagoras to form equation in $a$ | M1 | 3.4 | Use trigonometry or Pythagoras' theorem to form equation in $a$ |
| | $6.25=0.25\left((2a-4)^2+16\right)$<br>$\left(4a^2-16a+7=0\right)$ | A1ft<br>A1 | 1.1b<br>1.1b | Unsimplified equation with at most one error. Follow their $a\mathbf{i}$ Correct unsimplified equation |
| **Or** | $\lambda^2+\mu^2=\frac{25}{4}$ | M1 | | |
| | $\mathbf{I}=\lambda\mathbf{i}+\mu\mathbf{j}=\frac{1}{2}\left((x-4)\mathbf{i}-4\mathbf{j}\right)$ | M1 | | |
| | $\mu=-2$ | A1 | | Dependent on 2nd M (for impulse) |
| | $\lambda^2=\frac{9}{4}$ | A1 | | |

| | | | | |
|---|---|---|---|---|
| **Or** | Use of $\mathbf{I}=m(\mathbf{v}-\mathbf{u})$ to form vector triangle | M1 | 3.3 | |
| | Form equation in their $a$ | M1 | 3.4 | |
| | $6.25=a^2+8-2a\sqrt{8}\times\frac{1}{\sqrt{2}}$<br>$\left(4\times 6.25=b^2+32-2b\sqrt{32}\times\frac{1}{\sqrt{2}}\right.$<br>for velocity $b\mathbf{i}$$\left.\right)$<br>$\left(4a^2-16a+7=0\right)$ | A1ft<br>A1 | 1.1b<br>1.1b | |
| | $a=\frac{7}{2},\ \frac{1}{2}$<br>$\Rightarrow \mathbf{I}=\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns) | M1 | 1.1b | Complete correct method to solve to find a pair of values for λ and μ |
| | or<br>$\mathbf{I}=-\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns) | A1 | 1.1b | Two correct pairs of values for λ and μ |
| | or<br>$\mathbf{I}=-2\mathbf{i}-\frac{3}{2}\mathbf{j}$ (Ns) | M1 | 2.2a | Use symmetry in complete correct method to find one of the other pairs of values for λ and μ |
| | or<br>$\mathbf{I}=-2\mathbf{i}+\frac{3}{2}\mathbf{j}$ (Ns) | A1 | 1.1b | All four correct pairs<br>(They do not need to write out the impulse in full) |
| | | **(9)** | | |
| | | | | **(9 marks)** |

**Q4.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| a | Impulse-momentum equation | M1 | 3.1a |
| | $\mathbf{J}=0.5(-\mathbf{i}+6\mathbf{j}-4\mathbf{i}-3\mathbf{j})$ $(\mathbf{J}=0.5(-5\mathbf{i}+3\mathbf{j}))$ | A1 | 1.1b |
| | Find magnitude of **J**: | M1 | 1.1b |
| | $\lvert\mathbf{J}\rvert^2=\frac{1}{4}(25+9), \quad \lvert\mathbf{J}\rvert=\frac{\sqrt{34}}{2}$ (N s) | A1 | 1.1b |
| | | **(4)** | |
| b |   | | |
| | Correct use of trig | M1 | 3.1a |
| | $\alpha^\circ=180^\circ-\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{3}{5}$ or $\alpha^\circ=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{3}$ | A1ft | 1.1b |
| | $\alpha=112$ | A1 | 1.1b |
| | | (3) | |
| balt | Use scalar product of $\mu\mathbf{J}$ and $4\mathbf{i}+3\mathbf{j}$ to find the angle | M1 | 3.1a |
| | $\cos\alpha^\circ=\frac{-20+9}{\sqrt{34}\times5}$ | A1ft | 1.1b |
| | $\alpha=112$ | A1 | 1.1b |
| | | **(3)** | |
| balt | Use of cosine rule in triangle of momenta or equivalent | M1 | 3.1a |
| | $\alpha^\circ=180^\circ-\cos^{-1}\left(\frac{34+25-37}{2\times5\times\sqrt{34}}\right)$ | A1ft | 1.1b |
| | $\alpha=112$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

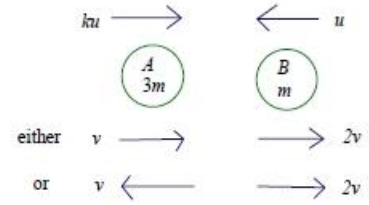
| Notes: | |
|---|---|
| (a)M1 | Dimensionally correct. Must be subtracting, but condone subtracting in the wrong order. |
| A1 | Correct unsimplified equation |
| M1 | Correct application of Pythagoras to find the magnitude. (from $\pm\mathbf{J}$) |
| A1 | 2.9 or better (2.9154….) (from $\pm\mathbf{J}$) |
| (b)M1 | Correct use of trig to find a relevant angle using $4\mathbf{i}+3\mathbf{j}$ and their $\mathbf{J}$<br>i.e. $\alpha^\circ$ or $180^\circ-\alpha^\circ$ Allow $\left\lvert\frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right\rvert$ |
| A1ft | Correct unsimplified expression for the required angle. Follow their $\mathbf{J}$ A0 for $\left\lvert\frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right\rvert$<br>Do not ISW |
| A1 | 110 or better (112.166…..) or accept 247.8....° |
| | |

**Q5.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | Use of Impulse = change in momentum | M1 | 3.1a |
| | $0.5(\mathbf{v}-8\mathbf{i})=\lambda(-\mathbf{i}+\mathbf{j})$<br>$(\mathbf{v}=(-2\lambda+8)\mathbf{i}+2\lambda\mathbf{j})$ | A1 | 1.1b |
| | Use of Pythagoras: | M1 | 3.1a |
| | e.g. $160=(-2\lambda+8)^2+(2\lambda)^2$<br>$(160=4\lambda^2-32\lambda+64+4\lambda^2)$ | A1 | 1.1b |
| | Form and solve quadratic in $\lambda$ : $8\lambda^2-32\lambda-96=0$<br>$(\lambda^2-4\lambda-12=(\lambda-6)(\lambda+2)=0)$ | M1 | 2.1 |
| | $\Rightarrow \lambda=6$ | A1 | 1.1b |
| | Find the required angle: $180°-\tan^{-1}3$ | M1 | 1.1b |
| | $\theta=108$ | A1 | 2.2a |
| | | (8) | |
| | | | (8 marks) |

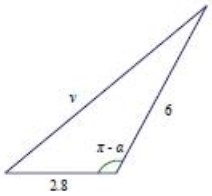
| Notes: | |
|---|---|
| M1 | Must be subtracting two values for momentum, but condone subtraction in the wrong order |
| A1 | Correct unsimplified equation |
| M1 | Correct use of final speed with their v |
| A1 | Correct unsimplified equation in one unknown or pair of simultaneous equations |
| M1 | Simplify and solve for $\lambda$ from correct working |
| A1 | Correct positive solution only |
| M1 | Complete method to solve for $\theta$ |
| A1 | 108 or better (108.4349….) |
| | |

**Q6.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| a |  | | |
| | Note that if they start with their 2v to the left this creates an impossible situation (the particles need to pass through each other). The maximum score is M1M1M1. | | |
| | Impulse received by $B$: | M1 | 3.4 |
| | $\frac{3}{2}mu = m\left(2v-\left(-u\right)\right)$ | A1 | 1.1b |
| | $v=\frac{u}{4}$ | A1 | 1.1b |
| | | (3) | |
| b | Use of CLM or Impulse-momentum for one option for $A$: | M1 | 3.4 |
| | $3kmu-mu=2mv+3mv\left(=\frac{5mu}{4}\right)$<br>or $3m\left(v-ku\right)=-\frac{3mu}{2}$ $\left(3mu\left(\frac{1}{4}+\frac{1}{2}\right)=3mku\right)$ | A1ft | 1.1b |
| | $k=\frac{3}{4}$ | A1 | 1.1b |
| | Form a second equation in $k$<br>$\left(3mku-mu=2mv-3mv\left(=-\frac{mu}{4}\right)\right.$ or $\left.3m\left(v+ku\right)=\frac{3mu}{2}\right)$ | M1 | 3.1a |
| | $k=\frac{1}{4}$ | A1 | 1.1b |
| | | (5) | |
| | | | |
| | | | **(T0tal 8 Marks)** |

| **Notes** | |
|---|---|
| (a)M1 | Form impulse-momentum equation for $B$ (or $A$).<br>May be expressed as either $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct.<br>Must be considering difference in velocities<br>Must have a correct combination of mass and velocity: pairing velocity of one with the mass of the other scores M0<br>Allow for subtraction the wrong way round or impulse in the wrong direction.<br>Assuming that you have not seen an incorrect formula stated, allow for $2v+u$ without overt evidence of subtraction.<br>Allow if the common factor of $m$ is not seen |
| A1 | Correct unsimplified equation for $B$ (or $A$).<br>Allow without $m$ |
| A1 | Correct answer only |
| (b) M1 | Correct method to form an equation in $k$. Must be dimensionally correct<br>Condone sign errors in CLM.<br>Allows marks for CLM equation here if seen in (a) and used correctly to find $k$ here.<br>Rules for impulse-momentum as above. M1 is available if they have not reversed the direction of the impulse. An equation which allows for the change in direction by using $\mathbf{u} - \mathbf{v}$ can score full marks.<br>Could be working with either option for the direction of motion of $A$ |
| A1ft | Correct unsimplified equation in $u$, $v$ or their $v$ |
| A1 | One correct solution<br>Be aware that a sign error in the impulse-momentum equation for $A$ can lead to a fortuitous answer. A fortuitous answer scores A0<br>(FYI the incorrect answers are $\frac{-7}{4}$ and $\frac{1}{4}$ ) |
| M1 | Correct method to form a second equation in $k$ (reversing the direction of motion of $A$) |
| A1 | Second correct solution |

**Q7.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| | Impulse momentum equation(s) | M1 | 3.1a |
| | $\begin{pmatrix} 3\times\cos\alpha \\ 3\times\sin\alpha \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_x - 2.8 \\ v_y \end{pmatrix}$ $\left(v_x = \frac{32}{5},\quad v_y = \frac{24}{5}\right)$ | A1<br>A1 | 1.1b<br>1.1b |
| | $v = \frac{1}{5}\sqrt{32^2 + 24^2}$ | M1 | 1.1b |
| | $= 8\left(\text{ms}^{-1}\right)$ | A1 | 1.1b |
| | Alternative working parallel and perpendicular to the impulse:<br>$\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_1 - 2.8\times\cos\alpha \\ v_2 \pm 2.8\times\sin\alpha \end{pmatrix}$ $v_1 = 7.68,\ v_2 = \pm 2.24$<br>$v = \sqrt{7.68^2 + 2.24^2} = 8\left(\text{ms}^{-1}\right)$ | | |
| | | (5) | |
| **alt** |  | | |
| | Using cosine rule: | M1 | |
| | $v^2 = 2.8^2 + 6^2 - 2\times 2.8\times 6\cos(\pi - \alpha)$ | A1<br>A1 | |
| | Solve for $v$ | M1 | |
| | $v = 8\left(\text{ms}^{-1}\right)$ | A1 | |
| | | (5) | |
| | | **(Total 5 marks)** | |

| Notes | |
|---|---|
| M1 | Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ in two dimensions. (i.e. resolving used) Dimensionally correct. Allow for a combined equation in vector format or for just one component. Condone sin/cos confusion.<br>Allow if $m$ seen but not substituted. |
| A1<br>A1 | Equation for one component correct unsimplified<br>Equations for both components correct unsimplified<br>Allow A1A1 for a correct unsimplified vector equation<br>Allow A marks if in terms of $m$ and $\alpha$ |
| M1 | Correct use of Pythagoras for their components to obtain the numerical value of the speed<br>This may be seen or implied: an alert candidate might spot the 3, 4, 5 triangle. |
| A1 | Correct only |
| Alt | |
| M1 | Correct use of cosine rule in a dimensionally correct triangle. The lengths of the sides must be consistent, i.e. v, 2.8 and 6 or $\frac{1}{2}v$, 1.4 and 3 and it must be a correct vector triangle (vectors combined correctly) |
| A1<br>A1 | Unsimpified equation with at most one error<br>Correct unsimplified equation |
| M1 | Substitute for trig. and solve for $v$ |
| A1 | Correct only |